PHYS 2212
Recitation 05 February 22–25

Example

A total charge Q is distributed uniformly throughout a spherical volume of radius R. Let r denote the distance of a point from the center of the sphere of charge.

a) Use Gauss's law to find the magnitude of the electric field at a point outside the sphere ($r > R$) and at a point inside the sphere ($r < R$).

b) Determine the electric potential at both the surface of the sphere and at the center of the sphere, with respect to zero at an infinite distance from the sphere. Express your algebraic answers in terms of Q, R, and fundamental constants.

PHYS 2212 Section: ______ Name: ______________________

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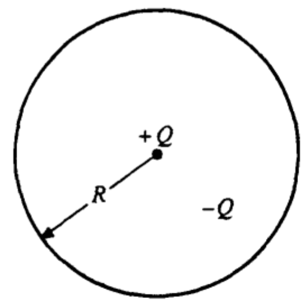
Problem

A negative charge $-Q$ is uniformly distributed throughout the spherical volume of radius R shown. A positive point charge $+Q$ is at the center of the sphere. Determine the electric field $\vec{E}$ both outside ($r > R$) and inside ($r < R$) the sphere. Determine the electric potential V both outside and inside the sphere, with respect to zero at an infinite distance from the sphere. Express your algebraic answers in terms of the parmeters defined in the problem and fundamental constants.



PHYS 2212 **Solution**
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a) $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$ $r > R$:

$E 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

$E 4\pi r^2 = \frac{1}{\epsilon_0} \rho V_{in} = \frac{1}{\epsilon_0}\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3$

$\Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}}$

b) $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$ ← integrate out so that $\vec{E}$ and $d\vec{s}$ both radially out

$\Rightarrow \vec{E} \cdot d\vec{s} = E\,dr$ $a = R$ $b = \infty$

$\cancel{V_\infty}^{\,0} - V(R) = -\int_R^\infty E\,dr \Rightarrow V_{sur} = V(R) = \int_R^\infty \frac{kQ}{r^2}\,dr$

$\boxed{V_{sur} = \frac{kQ}{R}}$ Note: For $r > R$ $V(r) = \frac{kQ}{r}$

$V_{sur} = V(r = R)$

For $r < R$, find $V(r)$ by integrating out $E(r) = \frac{kQ}{R^3}r$ from $r = r$ to $r = R$:

$V_R - V(r) = -\int_r^R E\,dr$, and $V_R = V_{sur} = \frac{kQ}{R}$

$\Rightarrow V(r) = V_{sur} + \int_r^R \frac{kQ}{R^3} r\,dr = \frac{kQ}{R} + \frac{kQ}{R^3}\left(\frac{1}{2}R^2 - \frac{1}{2}r^2\right)$

So for inside, $r < R$: $V(r) = \frac{kQ}{R} + \frac{kQ}{2R^3}(R^2 - r^2)$

At the center $r = 0$: $V_c = V(0) = \frac{kQ}{R} + \frac{kQ}{2R^3}(R^2 - 0)$

$\boxed{V_c = \frac{3kQ}{2R}}$

Note: Faster to show this answer by settings limits $r = 0 \rightarrow r = R$

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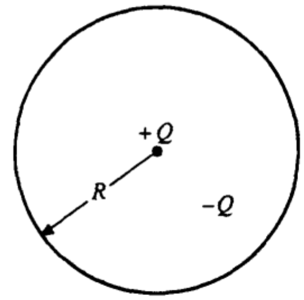

Electric Field: Outside

$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}, \quad Q_{in} = 0$$

$$\Rightarrow \boxed{E = 0}$$

Inside

$r < R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{1}{\epsilon_0}\left[Q + \frac{-Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3\right]$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}\left[1 - \frac{r^3}{R^3}\right]$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\left[1 - \left(\frac{r}{R}\right)^3\right]\hat{r}}$$

← dimensional analysis easy

Electric Potential: Outside

$r > R$

$$V_b - V_a = -\int_a^b \vec{E}^{\,\nearrow 0} \cdot d\vec{s}$$

$$V_\infty = 0, \quad E = 0 \quad \Rightarrow \boxed{V = 0}$$

$$V_R = V_{sur} = 0$$

(No work is required to move a test charge in the space around this sphere)

Inside

$r < R$

$$\cancel{V_R}^{\,\nearrow 0} - V_r = -\int_r^R E\, dr$$

← integrate outwards

$$\Rightarrow \quad V_r = \int_r^R \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right] dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r} - \frac{r^2}{2R^3}\right]_r^R$$

$$= \frac{Q}{4\pi\epsilon_0}\left[-\left(\frac{1}{R} - \frac{1}{r}\right) - \frac{1}{2R^3}\left(R^2 - r^2\right)\right]$$

$$\boxed{V_r = \frac{Q}{4\pi\epsilon_0}\left[\left(\frac{1}{r} - \frac{1}{R}\right) + \frac{(r^2 - R^2)}{2R^3}\right]}$$